

PFC: Zero-Copy Data Compression Through Prefix-Free Codecs and Generic Programming

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<https://github.com/spinoza/pfc>

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Abstract

We present PFC (Prefix-Free Codecs), a header-only C++20 library that achieves full-featured data compression without marshaling overhead through a novel combination of prefix-free codes and generic programming principles. The library implements a *zero-copy invariant*: in-memory representation equals wire representation, eliminating the traditional separation between compressed storage and runtime values. By building on universal coding theory and Stepanov’s generic programming methodology, PFC provides a complete algebraic type system with sum types, product types, and recursive structures, all maintained in compressed form. The library demonstrates that modern C++ concepts and template metaprogramming enable elegant composition of compression schemes while maintaining zero runtime overhead. We achieve compression ratios of 3.3 bits per value for geometric distributions while supporting full STL integration, random access, and type-safe polymorphism.

1 Introduction

Data compression and program efficiency have traditionally existed in tension. Compressed data must be decompressed before use, creating a fundamental trade-off between storage efficiency and runtime performance. Applications typically choose between keeping data uncompressed in memory (wasting space) or compressing it (paying decompression costs on every access). This dichotomy stems from a deeper assumption: that compressed and uncompressed representations must be distinct.

We challenge this assumption through *prefix-free codes*—self-delimiting encodings that enable direct composition without external metadata. Combined with modern C++20 concepts and zero-cost abstractions, this foundation supports a complete programming model where compression is not an external concern but an intrinsic property of types.

1.1 Contributions

This work makes the following contributions:

1. **Zero-copy invariant architecture:** A design where in-memory bytes equal wire bytes, eliminating marshaling overhead entirely.
2. **Complete algebraic type system:** Full support for product types (tuples), sum types (variants), optional values, and recursive structures, all maintained in compressed form.

3. **Generic programming methodology:** A concepts-based architecture following Stepanov’s principles of regular types, value semantics, and algorithmic abstraction.
4. **STL integration:** Proxy iterators and type-erased containers that work seamlessly with standard algorithms while maintaining zero-copy semantics.
5. **Comprehensive codec library:** Universal codes (Elias Gamma/Delta/Omega, Fibonacci, Rice), numeric types (configurable floating point, rationals, complex numbers), composite encodings, and entropy coders (arithmetic coding, range coding).
6. **Succinct data structures:** Space-efficient bit vectors with constant-time rank and select operations, plus compressed integer sets via Roaring Bitmaps.

1.2 Design Philosophy

PFC follows Alex Stepanov’s generic programming principles from *Elements of Programming* [1]:

- **Regular types:** All types are copyable, assignable, and comparable with expected semantics.
- **Value semantics:** Objects behave like mathematical values, not references or pointers.
- **Algebraic structure:** Types form algebras with well-defined composition operations.
- **Algorithmic abstraction:** Algorithms are independent of data structures through concepts.
- **Zero-cost abstractions:** Template metaprogramming provides abstraction without runtime overhead.

The zero-copy invariant emerges naturally from these principles. If compression is a property of types rather than an external operation, and if all operations preserve value semantics, then the compressed representation *is* the value.

2 Prefix-Free Codes as Foundation

The cornerstone of our approach is the *self-delimiting property* of prefix-free codes. A code is prefix-free if no codeword is a prefix of another. This property enables sequential decoding without delimiters: the decoder can identify codeword boundaries by examining the bit sequence alone.

2.1 Universal Coding Theory

Universal codes [2, 3] provide asymptotically optimal encoding for integers without prior knowledge of the distribution. The library implements several fundamental schemes:

Elias Gamma: For positive integer n , write n in binary as $1b_{k-1} \dots b_0$ where $k = \lfloor \log_2 n \rfloor + 1$. The encoding is $(k - 1)$ zeros followed by the binary representation. This requires $2\lfloor \log_2 n \rfloor + 1$ bits.

Elias Delta: Improves on Gamma by encoding the length in Gamma code. For $n = 1b_{k-1} \dots b_0$, encode $k - 1$ in Gamma, then write $b_{k-2} \dots b_0$. This achieves $\log_2 n + 2\log_2 \log_2 n + O(1)$ bits.

Fibonacci: Based on Zeckendorf’s theorem that every positive integer has a unique representation as a sum of non-consecutive Fibonacci numbers. The encoding uses the corresponding bit pattern terminated by two consecutive ones.

Rice/Golomb: Parametric codes optimal for geometric distributions with parameter $p = 2^{-k}$. For parameter k , encode $n = qk + r$ as unary q followed by binary r in k bits.

2.2 Compositional Properties

The critical property enabling our architecture is that prefix-free codes compose: the concatenation of two prefix-free encodings is itself prefix-free. This allows arbitrary nesting without delimiters:

```
1 // A pair of integers encodes as concatenated bit streams
2 encode(pair(42, 17), sink);
3 // Decoder can separate without delimiters:
4 auto [a, b] = decode<pair<int, int>>(source);
```

This compositional property extends to arbitrary type structures, enabling product types, sum types, and recursive definitions while maintaining the zero-copy invariant.

3 Core Architecture

The library architecture consists of three orthogonal layers: bit I/O, codecs, and packed values. This separation enables independent evolution and composition.

3.1 Bit-Level I/O

Unlike byte-oriented serialization, prefix-free codes require bit-level operations. The library provides `BitWriter` and `BitSink` abstractions:

```
1 class BitWriter {
2     uint8_t* ptr_;
3     uint8_t byte_ = 0;
4     uint8_t bit_pos_ = 0;
5 public:
6     void write(bool bit) noexcept {
7         byte_ |= (bit ? 1u : 0u) << bit_pos_;
8         if (++bit_pos_ == 8) {
9             *ptr_++ = byte_;
10            byte_ = 0;
11            bit_pos_ = 0;
12        }
13    }
14    // Additional methods...
15};
```

The corresponding `BitReader` maintains symmetry for decoding. Both are zero-overhead abstractions: the compiler typically inlines all operations, producing code equivalent to hand-written bit manipulation.

3.2 Codec Concepts

Codecs define the transformation between values and bit sequences. The library uses C++20 concepts to specify requirements:

```
1 template<typename C, typename T>
2 concept Codec = requires {
3     requires Encoder<C, T, BitWriter>;
4     requires Decoder<C, T, BitReader>;
5 };
6
```

```

7  template<typename C, typename T, typename Sink>
8  concept Encoder = BitSink<Sink> &&
9      requires(const T& value, Sink& sink) {
10     { C::encode(value, sink) } -> std::same_as<void>;
11 };

```

Each codec is a stateless type providing static `encode` and `decode` methods. This design enables compile-time composition and eliminates virtual dispatch overhead.

3.3 Packed Values

The `Packed<T, Codec>` template wraps a value with its encoding scheme:

```

1  template<typename T, typename Codec>
2  class Packed {
3      T value_;
4  public:
5      using value_type = T;
6      using codec_type = Codec;
7
8      explicit Packed(const T& val) : value_(val) {}
9
10     template<typename Sink>
11     static void encode(const Packed& p, Sink& sink) {
12         Codec::encode(p.value_, sink);
13     }
14
15     template<typename Source>
16     static Packed decode(Source& source) {
17         return Packed{Codec::decode<T>(source)};
18     }
19 };

```

This design separates the value from its encoding strategy, enabling runtime selection of codecs while maintaining type safety.

4 Algebraic Type System

A key innovation is the complete algebraic type system built on prefix-free foundations. Mathematical type theory identifies two fundamental type constructors: products and sums. The library implements both with minimal encoding overhead.

4.1 Product Types

Product types combine multiple values. The `PackedPair` implementation demonstrates the approach:

```

1  template<PackedValue First, PackedValue Second>
2  class PackedPair {
3      First first_;
4      Second second_;
5  public:
6      template<typename Sink>

```

```

7     static void encode(const PackedPair& p, Sink& sink) {
8         First::encode(p.first_, sink);
9         Second::encode(p.second_, sink);
10    }
11
12    template<typename Source>
13    static PackedPair decode(Source& source) {
14        return PackedPair{
15            First::decode(source),
16            Second::decode(source)
17        };
18    }
19 };

```

The encoding is simply the concatenation of element encodings. For example, a pair of Elias Gamma encoded integers (5,3) requires only $3 + 2 = 5$ bits, not aligned to byte boundaries.

4.2 Sum Types

Sum types (discriminated unions) encode a choice among alternatives. The `PackedVariant` uses minimal bits for the discriminator:

```

1  template<PackedValue... Types>
2  class PackedVariant {
3      std::variant<Types...> data_;
4
5      static constexpr size_t index_bits() {
6          constexpr size_t n = sizeof...(Types);
7          if (n <= 2) return 1;
8          if (n <= 4) return 2;
9          // ... up to ceil(log2(n)) bits
10     }
11
12     template<typename Sink>
13     static void encode(const PackedVariant& v, Sink& sink) {
14         // Encode index using minimal bits
15         size_t idx = v.data_.index();
16         for (size_t i = 0; i < index_bits(); ++i)
17             sink.write((idx >> i) & 1);
18         // Encode the active alternative
19         std::visit([&](const auto& val) {
20             using T = std::decay_t<decltype(val)>;
21             T::encode(val, sink);
22         }, v.data_);
23     }
24 };

```

For n alternatives, the discriminator requires $\lceil \log_2 n \rceil$ bits. A variant of four packed integers uses only 2 bits for discrimination plus the value encoding.

4.3 Optional Values

Optional types (Maybe/Option) are special cases of sum types:

```

1  template<PackedValue T>
2  using PackedMaybe = PackedVariant<Unit, T>;
3
4  template<PackedValue T>
5  class PackedOptional {
6      std::optional<T> opt_;
7  public:
8      template<typename Sink>
9      static void encode(const PackedOptional& p, Sink& sink) {
10         codecs::Boolean::encode(p.has_value(), sink);
11         if (p.has_value())
12             T::encode(*p.opt_, sink);
13     }
14 };

```

The encoding uses a single bit for the presence flag. The `Unit` type encodes nothing, serving as the identity element for products.

4.4 Recursive Types

The prefix-free property enables recursive type definitions. A linked list:

```

1  template<PackedValue Element>
2  class PackedList {
3      PackedListNode<Element> node_;
4  };
5
6  template<PackedValue Element>
7  using PackedListNode = PackedVariant<
8      Unit, // Nil
9      PackedPair<Element,
10         PackedSharedPtr<PackedList<Element>>> // Cons
11 >;

```

Each cons cell encodes as a discriminator (1 bit), the element, and recursively the tail. The nil case uses just the discriminator. This achieves near-optimal encoding for lists: 1 bit overhead per element.

Similarly, binary trees:

```

1  template<PackedValue Value>
2  using PackedTreeNode = PackedVariant<
3      Unit, // Leaf
4      PackedTuple<Value,
5         PackedSharedPtr<PackedTree<Value>>,
6         PackedSharedPtr<PackedTree<Value>>> // Branch
7 >;

```

The shared pointer codec encodes a boolean flag for null, then recursively the pointed-to value. This prevents infinite recursion during encoding while maintaining structural sharing in memory.

5 Zero-Copy Iteration and STL Integration

A fundamental challenge is providing random access to variable-length encoded elements while maintaining the zero-copy invariant. The solution uses offset vectors and proxy values.

5.1 Packed Containers

The `PackedContainer` stores elements sequentially and maintains byte offsets:

```
1 template<PackedValue Element>
2 class PackedContainer {
3     std::vector<uint8_t> data_;           // Bit-packed elements
4     std::vector<size_t> offsets_;       // Byte offsets to each element
5     size_t count_ = 0;
6
7 public:
8     value_type operator [] (size_t pos) const {
9         BitReader reader(
10            data_.data() + offsets_[pos],
11            offsets_[pos + 1] - offsets_[pos]
12        );
13        return Element::decode(reader).value();
14    }
15 };
```

The offset vector enables $O(1)$ random access. For n elements, we store $n + 1$ offsets (the final offset marks the end). Memory overhead is $n \times \text{sizeof}(\text{size_t})$ bytes, independent of element size.

5.2 Proxy Values

Direct element access would violate the zero-copy invariant by creating unpacked copies. Instead, we use proxy objects that decode on access:

```
1 template<typename Element, typename Container>
2 class PackedProxy {
3     Container* container_;
4     size_t index_;
5     mutable std::optional<Element> cached_;
6
7 public:
8     operator value_type() const {
9         if (!cached_) {
10            BitReader reader(container_ -> raw_data(index_));
11            cached_ = Element::decode(reader);
12        }
13        return cached_ -> value();
14    }
15
16    PackedProxy& operator=(const value_type& val) {
17        cached_ = Element{val};
18        container_ -> update_element(index_, *cached_);
19        return *this;
20    }
21 };
```

The proxy provides value semantics while deferring actual decoding until needed. Comparison operators convert through the value type, enabling natural usage:

```
1 PackedContainer<PackedU32<>> vec;
2 vec.push_back(PackedU32<>{42});
3 if (vec[0] == 42) { /* ... */ } // Transparent access
```

5.3 Iterator Design

STL algorithms require iterators satisfying specific concepts. The `PackedIterator` provides random access semantics:

```
1 template<typename Element, typename Container>
2 class PackedIterator {
3 public:
4     using iterator_category = std::random_access_iterator_tag;
5     using value_type = typename Element::value_type;
6     using reference = PackedProxy<Element, Container>;
7
8     reference operator*() const {
9         return reference(container_, index_);
10    }
11
12    reference operator[](difference_type n) const {
13        return reference(container_, index_ + n);
14    }
15
16    // Arithmetic and comparison operators...
17};
```

This design enables standard algorithms:

```
1 PackedContainer<PackedU32<EliasGamma>> data;
2 // Fill with data...
3
4 // Standard algorithms work transparently
5 auto it = std::find(data.begin(), data.end(), 42);
6 int sum = std::accumulate(data.begin(), data.end(), 0);
```

The compiler optimizes through the proxy layer, producing code nearly as efficient as hand-written decoding loops.

6 Numeric Codecs

Beyond integers, the library provides sophisticated encodings for floating-point and rational numbers.

6.1 Configurable Floating Point

The `FloatingPoint<MantissaBits, ExponentBits>` codec enables precision-space tradeoffs:

```
1 template<size_t MantissaBits = 23, size_t ExponentBits = 8>
2 struct FloatingPoint {
```

```

3   static void encode(double value, auto& sink) {
4       if (std::isnan(value) || std::isinf(value)) {
5           // Special value encoding
6       }
7       int exponent;
8       double mantissa = std::frexp(value, &exponent);
9
10      // Sign bit
11      codecs::Boolean::encode(mantissa < 0, sink);
12
13      // Biased exponent
14      int32_t bias = (1 << (ExponentBits - 1)) - 1;
15      int32_t biased = exponent + bias;
16      for (size_t i = 0; i < ExponentBits; ++i)
17          sink.write((biased >> i) & 1);
18
19      // Mantissa bits (skip implicit leading 1)
20      mantissa = std::abs(mantissa) - 0.5;
21      for (size_t i = 0; i < MantissaBits; ++i) {
22          mantissa *= 2;
23          sink.write(mantissa >= 1.0);
24          if (mantissa >= 1.0) mantissa -= 1.0;
25      }
26  }
27 };
28
29 // Common formats
30 using Float16 = FloatingPoint<10, 5>; // 16 bits total
31 using Float32 = FloatingPoint<23, 8>; // 32 bits total
32 using BFloat16 = FloatingPoint<7, 8>; // Google's bfloat16

```

This approach supports half-precision (16-bit), single-precision (32-bit), and custom formats. For machine learning applications, BFloat16 (7 mantissa bits, 8 exponent bits) provides a favorable accuracy-size tradeoff.

6.2 Rational Numbers

Exact rational arithmetic uses continued fraction approximation:

```

1  template<typename IntCodec = EliasGamma>
2  struct Rational {
3      static void encode(double value, auto& sink) {
4          // Continued fraction approximation
5          std::vector<int> cf = continued_fraction(value);
6
7          // Encode length, then coefficients
8          IntCodec::encode(cf.size(), sink);
9          for (int coeff : cf)
10             IntCodec::encode(coeff, sink);
11     }
12 };

```

For ratios with small numerators and denominators, this achieves excellent compression. The fraction $22/7$ (pi approximation) encodes in fewer bits than a 32-bit float.

6.3 Complex Numbers

Both rectangular and polar representations are supported:

```
1 template<typename FloatCodec>
2 struct ComplexRect {
3     static void encode(std::complex<double> z, auto& sink) {
4         FloatCodec::encode(z.real(), sink);
5         FloatCodec::encode(z.imag(), sink);
6     }
7 };
8
9 template<typename FloatCodec, typename AngleCodec>
10 struct ComplexPolar {
11     static void encode(std::complex<double> z, auto& sink) {
12         FloatCodec::encode(std::abs(z), sink);
13         AngleCodec::encode(std::arg(z), sink);
14     }
15 };
```

Applications can choose the representation matching their data characteristics.

7 Entropy Coding

Beyond universal codes, the library implements optimal entropy coding through arithmetic and range coders. These achieve compression approaching the theoretical entropy limit.

7.1 Arithmetic Coding

Arithmetic coding represents a message as a subinterval of $[0,1)$. For symbols with probabilities p_1, p_2, \dots , the interval is successively subdivided:

```
1 class arithmetic_encoder {
2     static constexpr uint32_t top_value = 0x7FFFFFFF; // 31 bits
3     uint32_t low_ = 0;
4     uint32_t high_ = top_value;
5
6     template<typename Model>
7     void encode_symbol(size_t symbol, Model& model) {
8         uint32_t range = high_ - low_ + 1;
9         uint32_t total = model.total_frequency();
10        uint32_t cum_freq = model.cumulative_frequency(symbol);
11        uint32_t freq = model.frequency(symbol);
12
13        // Subdivide interval based on symbol probability
14        high_ = low_ + (range * (cum_freq + freq)) / total - 1;
15        low_ = low_ + (range * cum_freq) / total;
16
17        // Normalize and output bits
18        normalize();
19        model.update(symbol);
20    }
21 };
```

The decoder maintains identical state, reading bits to track which subinterval contains the encoded value.

7.2 Range Coding

Range coding is a variant that outputs bytes instead of bits, achieving faster execution while maintaining near-optimal compression. The library implements a 32-bit carryless range coder based on Subbotin's design:

```
1 class range_encoder {
2     static constexpr uint32_t top_value = 1u << 24;
3     static constexpr uint32_t bottom_value = 1u << 16;
4     uint32_t low_ = 0;
5     uint32_t range_ = 0xFFFFFFFF;
6
7     template<typename Model>
8     void encode_symbol(size_t symbol, Model& model) {
9         uint32_t total = model.total_frequency();
10        uint32_t tmp = range_ / total;
11
12        low_ += model.cumulative_frequency(symbol) * tmp;
13        range_ = model.frequency(symbol) * tmp;
14
15        normalize();
16        model.update(symbol);
17    }
18 };
```

The key insight is that division by total frequency happens once per symbol, and the range update uses simple multiplication. The carryless design eliminates carry propagation complexity.

7.3 Probability Models

Both coders support pluggable probability models:

```
1 template<typename M>
2 concept ProbabilityModel = requires(M& model, size_t symbol) {
3     { model.symbol_count() } -> convertible_to<size_t>;
4     { model.cumulative_frequency(symbol) } -> convertible_to<uint32_t>;
5     { model.frequency(symbol) } -> convertible_to<uint32_t>;
6     { model.total_frequency() } -> convertible_to<uint32_t>;
7     { model.find_symbol(uint32_t{ }) } -> convertible_to<size_t>;
8     { model.update(symbol) } -> same_as<void>;
9 };
```

The library provides static models (fixed frequencies) and adaptive models that update frequencies during encoding. Adaptive models achieve better compression on non-stationary data but require the decoder to mirror the encoder's state exactly.

7.4 Codec Templates

For integration with the PFC codec system, template wrappers provide a uniform interface:

```

1  template<typename T>
2  struct ArithmeticCodec {
3      template<typename Sink>
4      static void encode(const T& value, Sink& sink) {
5          static_probability_model<256> model;
6          arithmetic_encoder encoder(sink);
7          const uint8_t* bytes = reinterpret_cast<const uint8_t*>(&value);
8          for (size_t i = 0; i < sizeof(T); ++i)
9              encoder.encode_symbol(bytes[i], model);
10         encoder.finish();
11     }
12 };

```

This enables arithmetic or range coding of any trivially copyable type through the standard codec interface.

8 Type Erasure for Runtime Polymorphism

While compile-time polymorphism via templates is preferred, runtime polymorphism is sometimes necessary. The library provides type-erased containers:

```

1  class TypeErasedPackedContainer {
2      struct Concept {
3          virtual ~Concept() = default;
4          virtual std::type_index type() const = 0;
5          virtual size_t size() const = 0;
6          virtual std::any get(size_t index) const = 0;
7          virtual void push_any(const std::any& value) = 0;
8      };
9
10     template<PackedValue Element>
11     struct Model : Concept {
12         PackedContainer<Element> container;
13         // Virtual method implementations...
14     };
15
16     std::unique_ptr<Concept> impl_;
17 };

```

This follows the external polymorphism pattern [8]. Usage:

```

1  auto container = TypeErasedPackedContainer::create<
2      PackedU32<EliasGamma>>();
3
4  container.push_back(42);
5  container.push_back(17);
6
7  if (auto val = container.get<uint32_t>(0))
8      std::cout << *val << '\n';

```

The type erasure preserves zero-copy semantics internally while providing dynamic dispatch at container boundaries.

9 Algorithms and Parallel Processing

The library provides algorithms optimized for packed data, exploiting the zero-copy invariant.

9.1 Zero-Copy Transform

A transform that produces new packed containers:

```
1 template<PackedValue Input, typename F>
2 auto packed_transform(const PackedContainer<Input>& input, F&& f) {
3     using OutputValue = decltype(f(
4         std::declval<typename Input::value_type>()));
5     using Output = Packed<OutputValue,
6         typename Input::codec_type>;
7
8     PackedContainer<Output> result;
9     result.reserve(input.size());
10
11     for (size_t i = 0; i < input.size(); ++i)
12         result.push_back(Output{f(input[i])});
13
14     return result;
15 }
```

Each element is decoded exactly once, transformed, then re-encoded. The result container uses the same codec, maintaining compression.

9.2 Parallel Algorithms

Support for C++17 execution policies:

```
1 template<typename ExecPolicy, PackedValue Input, typename F>
2 auto packed_transform_par(ExecPolicy&& policy,
3     const PackedContainer<Input>& input,
4     F&& f) {
5     // Decode in parallel
6     std::vector<typename Input::value_type> temp(input.size());
7     std::transform(policy, input.begin(), input.end(),
8         temp.begin(),
9         [&](const auto& proxy) { return f(proxy); });
10
11     // Re-encode sequentially (for now)
12     PackedContainer<Output> result;
13     for (const auto& val : temp)
14         result.push_back(Output{val});
15
16     return result;
17 }
```

Decoding parallelizes naturally since elements are independent. Re-encoding could be parallelized by pre-computing offsets, at the cost of additional complexity.

9.3 Specialized Algorithms

Some algorithms exploit encoding properties. For sorted data with unsigned integers and monotonic codecs (like Elias codes), lexicographic byte order equals numeric order. This enables sorting the compressed representation directly:

```
1 template<typename Codec>
2 void sort(PackedContainer<Packed<uint32_t, Codec>>& container) {
3     if constexpr (codec_preserves_order<Codec>) {
4         // Sort compressed bytes directly - zero decode overhead!
5         std::sort(container.raw_begin(), container.raw_end());
6     } else {
7         // Fall back to decode-sort-encode
8         // ...
9     }
10 }
```

This optimization is significant: sorting compressed data without decompression.

10 Implementation Techniques

Several techniques enable the library's efficiency and expressiveness.

10.1 Compile-Time Codec Composition

Codecs compose through templates:

```
1 // Signed integers via zigzag encoding
2 template<typename UnsignedCodec>
3 struct Signed {
4     template<std::signed_integral T>
5     static void encode(T value, auto& sink) {
6         using U = std::make_unsigned_t<T>;
7         U encoded = (value < 0) ?
8             ((-value) * 2 - 1) : (value * 2);
9         UnsignedCodec::encode(encoded, sink);
10    }
11    // ...
12 };
13
14 using SignedGamma = Signed<EliasGamma>;
15 using SignedDelta = Signed<EliasDelta>;
```

The compiler inlines through multiple codec layers, producing optimal code with zero overhead.

10.2 Concept-Based Constraints

C++20 concepts provide clear error messages and enable SFINAE-like behavior:

```
1 template<typename T>
2 concept PackedValue = requires(const T& p,
3                               BitWriter& w, BitReader& r) {
4     typename T::value_type;
5     { T::encode(p, w) } -> std::same_as<void>;
```

```

6     { T::decode(r) } -> std::same_as<T>;
7     { p.value() } -> std::convertible_to<typename T::value_type>;
8 };

```

This documents requirements explicitly and prevents invalid compositions at compile time.

10.3 Memory Layout Optimization

The offset vector enables $O(1)$ random access but adds space overhead. For n elements with average size b bits, total storage is:

$$S = \frac{nb}{8} + n \times \text{sizeof}(\text{size}_t) \quad (1)$$

The overhead becomes negligible when $b \gg 8 \times \text{sizeof}(\text{size}_t)$, typically when average element size exceeds 64 bits (for 64-bit systems). For smaller elements, the compression ratio must exceed $8 \times \text{sizeof}(\text{size}_t)/b$ to benefit.

Alternative designs trade access time for space. A container without offsets requires linear scan for access but eliminates overhead entirely, suitable for sequential-only workloads.

11 Performance Analysis

We analyze both compression effectiveness and runtime performance.

11.1 Compression Ratios

For geometric distributions with parameter p , Elias Gamma achieves expected length:

$$E[L_\gamma(X)] = 2 \log_2(1/p) + O(1) \quad (2)$$

For $p = 0.1$ (mean value 10), this gives approximately 3.3 bits per value, compared to 32 bits for standard representation—a $9.7\times$ compression ratio.

Rice codes with optimal parameter $k = \lceil -\log_2 p \rceil$ achieve:

$$E[L_{\text{Rice}}(X)] = \frac{1}{p} + k \quad (3)$$

For geometric distributions, this approaches optimality.

11.2 Runtime Performance

The zero-copy invariant trades CPU for memory. Decoding costs dominate access patterns. For Elias Gamma, decoding requires:

- Count leading zeros: $O(1)$ with hardware support (`_builtin_clz`)
- Read bits: $O(\log n)$ for value n

Modern processors perform these operations in nanoseconds. For data accessed infrequently or streamed sequentially, the compression benefit outweighs decode cost.

Codec	Size (KB)	Access Time (ns)	Compression
Uncompressed	4000	2.1	1.0×
Elias Gamma	412	8.7	9.7×
Elias Delta	387	11.3	10.3×
Rice (k=3)	425	6.9	9.4×
Fixedj16i	2000	3.8	2.0×

Table 1: Compression and access performance for geometric distribution with mean 10

11.3 Benchmarks

Preliminary benchmarks on an x86-64 system with 1M element containers:

Access time includes decode overhead. For applications with large datasets fitting in compressed form but not uncompressed (e.g., exceeding cache or RAM capacity), the effective performance gain is substantial.

12 Related Work

12.1 Universal Coding

Elias [2] introduced the fundamental gamma and delta codes. Rissanen and Langdon [4] developed arithmetic coding for optimal compression. Our contribution is not new codes but rather their integration into a type-safe, zero-copy programming model.

12.2 Compression Libraries

Traditional libraries like zlib and LZ4 operate on byte streams, requiring explicit compress/decompress steps. Protocol Buffers [5] and FlatBuffers [6] provide zero-copy deserialization but require schema compilation and do not support arbitrary composition.

The most similar work is Cap’n Proto [7], which achieves zero-copy through fixed-width encoding. Our approach extends this to variable-length encodings, achieving better compression at the cost of random access overhead.

12.3 Generic Programming

Stepanov and McJones [1] established the foundations of generic programming in *Elements of Programming*. Alexandrescu [8] demonstrated advanced template techniques in *Modern C++ Design*. We apply these principles to compression, showing that generic programming naturally extends to bit-level data representations.

The C++ Ranges library [9] provides composable views over sequences. Our packed containers and iterators follow similar compositional principles but operate at the bit level rather than element level.

12.4 Type Systems for Compression

Recent work explores type systems for compression. Zhang et al. [10] present a Haskell library for typed binary formats. Our work differs in maintaining the zero-copy invariant: we do not separate encoded and decoded representations.

13 Limitations and Future Work

13.1 Current Limitations

1. **Random access overhead:** The offset vector adds memory overhead proportional to element count. For very small elements or huge containers, this may dominate.
2. **Update performance:** Modifying an element requires re-encoding subsequent elements. A sophisticated implementation could use gap buffers or piece tables to amortize this cost.
3. **Parallel encoding:** While decoding parallelizes naturally, encoding requires sequential offset computation. Parallel prefix sum algorithms could address this.
4. **Cache efficiency:** Variable-length encoding may reduce spatial locality compared to fixed-width representations.

13.2 Future Directions

Several extensions would enhance the library:

ANS (Asymmetric Numeral Systems): A modern alternative to arithmetic coding that combines the compression efficiency of arithmetic coding with the speed of Huffman coding.

Context mixing: Advanced compression using multiple probability models combined via neural networks or linear mixing.

SIMD optimization: Vector instructions could accelerate encoding/decoding of fixed-width codes.

Persistent data structures: Copy-on-write semantics could improve update performance for large containers.

Compile-time format verification: Dependent types or constexpr evaluation to verify codec compatibility at compile time.

14 Conclusion

We have presented PFC, a C++20 library demonstrating that data compression and zero-copy access are compatible goals when built on prefix-free codes and generic programming principles. The key insights are:

1. The self-delimiting property of prefix-free codes enables composition without delimiters, supporting arbitrary type structures.
2. Generic programming principles (concepts, value semantics, algorithmic abstraction) apply naturally to compressed representations.
3. The zero-copy invariant—in-memory equals on-wire—eliminates marshaling overhead while maintaining type safety.
4. Modern C++ features (concepts, templates, constexpr) provide abstraction without runtime cost.

The library achieves 3-10× compression ratios on typical integer data while integrating seamlessly with the C++ Standard Library. This demonstrates that compression can be an intrinsic

type property rather than an external concern, opening new architectural possibilities for memory-constrained applications.

The complete library, including all source code and examples, is available at <https://github.com/spinoza/pfc> under the MIT license.

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