Alea: A Modern C++ Library for Algebraic Random Elements

Technical Report
Alea Development Team
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Abstract

We present `alea`, a C++20 header-only library that implements probability distributions and Monte Carlo methods using generic programming and type erasure. The library provides: (1) a type-erased interface for probability distributions that enables runtime polymorphism while maintaining type safety; (2) a fluent API for distribution composition using established builder pattern techniques; (3) Monte Carlo integration with parallel execution; (4) SIMD-accelerated batch sampling for selected distributions when compiled with appropriate flags; and (5) comprehensive implementations of common continuous and discrete distributions. We evaluate `alea` against existing C++ libraries (Boost.Random, GSL, and standard library implementations), demonstrating comparable performance with improved ease of use. Through case studies in financial modeling, A/B testing, and quality control, we illustrate the library's practical applicability. While building on well-established techniques, `alea` contributes a cohesive, modern C++ interface that simplifies probabilistic programming tasks.

Keywords: Random elements, probabilistic programming, C++ templates, Monte Carlo methods, SIMD vectorization, type erasure, generic programming

1 Introduction

The modeling of random phenomena is fundamental to numerous domains including finance, engineering, machine learning, and scientific computing.

The C++ standard library and existing third-party libraries provide various levels of support for random number generation and statistical distributions. However, integrating these capabilities into complex applications often requires significant boilerplate code and careful management of type hierarchies.

The `alea` library addresses these practical challenges by providing a unified interface for probability distributions using modern C++ techniques. Building on established patterns from the literature [1], we apply type erasure to enable runtime polymorphism for distributions while maintaining static type safety. This approach simplifies common tasks such as mixing distributions, implementing Monte Carlo methods, and building hierarchical models.

1.1 Motivation

Modern applications increasingly require sophisticated probabilistic modeling capabilities:

- **Financial Risk Management**: Modeling portfolio returns with heavy-tailed distributions, copula-based dependencies, and regime-switching dynamics.
- **Machine Learning**: Implementing probabilistic models, Bayesian inference, and uncertainty quantification.
- **Scientific Computing**: Monte Carlo integration, MCMC sampling, and stochastic differential equations.
- **Quality Control**: Statistical process control, reliability analysis, and acceptance sampling.

Existing C++ libraries like Boost.Random provide excellent low-level primitives but lack the high-level abstractions needed for complex probabilistic modeling. Libraries from other languages (NumPy, R, Julia) offer rich statistical capabilities but cannot match C++'s performance requirements in production systems.

1.2 Contributions

This work makes the following contributions:

1. **Unified Distribution Interface**: We implement a type-erased interface for probability distributions that enables runtime polymorphism while preserving type safety, building on established type erasure patterns [1].

2. **Implementation Completeness**: We provide implementations of 25+ continuous and discrete distributions with optimized sampling algorithms selected based on parameter ranges, following algorithms from [2].

3. **Fluent API Design**: We implement a builder pattern-based API for distribution composition, evaluated through usability comparisons with existing libraries.

4. **Performance Optimization**: When compiled with appropriate flags, the library provides SIMD-accelerated batch sampling for uniform and normal distributions, achieving 3.6× speedup on AVX2 hardware.

5. **Empirical Validation**: We validate correctness through statistical tests (Kolmogorov-Smirnov, Anderson-Darling) and benchmark against three existing libraries, with results showing performance within 10% of specialized implementations.

1.3 Design Principles

`alea` follows these design principles:

Template-Based Efficiency: Core distribution implementations use templates to enable compile-time optimization, with measured overhead of less than 5% compared to direct standard library calls.

Composability: Distributions can be composed using standard functional patterns, enabling construction of mixture models and hierarchical distributions.

Error Handling: Parameter validation uses exceptions for runtime errors, with compile-time checks where possible using C++20 concepts.

Standard Library Integration: The library follows STL conventions for random number generators, accepting any UniformRandomBitGenerator as defined in [rand.req.urng].

2 Architecture

The `alea` library architecture is organized around three core abstractions: random processes, random elements, and type-erased distributions. These

abstractions work together to provide both mathematical rigor and practical efficiency.

2.1 Core Concepts

2.1.1 Random Process

A random process in `alea` is defined as a sequence of random elements $\{W_t\}$ indexed by time or another parameter. The fundamental requirement is a `sample(URBG&)` method:

```
template<typename T>
concept RandomProcess = requires(T t, std::mt19937& g) {
    typename T::value_type;
    { t.sample(g) } -> std::convertible_to<
        typename T::value_type>;
};
```

Listing 1: Random Process Concept

This C++20 concept [3] enables compile-time verification of interface requirements. The `sample` method represents the transformation from uniform random bits to the target distribution.

2.1.2 Random Element

Following [4], we define a random element as a measurable function from a probability space $(\Omega, \mathcal{F}, P)$ to a measurable space $(S, \mathcal{S})$. In the context of our C++ implementation, S represents the type `T` in `random_element<T>`, and measurability is implicitly satisfied through the type system. The implementation provides:

```
template <typename T>
class random_element {
public:
    using value_type = T;

    template <typename X>
    random_element(X x) :
        self_(make_shared<model<X>>(move(x))) {}

    template <typename URBG>
    T sample(URBG& gen) {
        return self_->sample(gen);
    }

    double pdf(const T& x) const {
```

```
        return self_->pdf(x);
    }

    double cdf(const T& x) const {
        return self_->cdf(x);
    }
};
```

Listing 2: Random Element Type Erasure

2.1.3 Type Erasure Implementation

We employ the external polymorphism pattern [1] to provide value semantics for distributions. This well-established technique allows runtime polymorphism without inheritance hierarchies:

```
template <typename T>
struct concept_base {
    virtual ~concept_base() = default;
    virtual T sample(std::mt19937&) = 0;
    virtual double pdf(const T&) const = 0;
    virtual double cdf(const T&) const = 0;
};

template <typename T, typename X>
struct model : concept_base<T> {
    X data;
    explicit model(X x) : data(move(x)) {}

    T sample(std::mt19937& gen) override {
        return data.sample(gen);
    }

    double pdf(const T& x) const override {
        if constexpr (has_pdf_v<X>) {
            return data.pdf(x);
        } else {
            // Monte Carlo estimation as fallback
            // Implementation details omitted for brevity
            return 0.0; // Placeholder
        }
    }
};
```

Listing 3: Type Erasure Pattern

2.2 Distribution Hierarchy

`alea` provides a comprehensive hierarchy of probability distributions organized by mathematical properties:

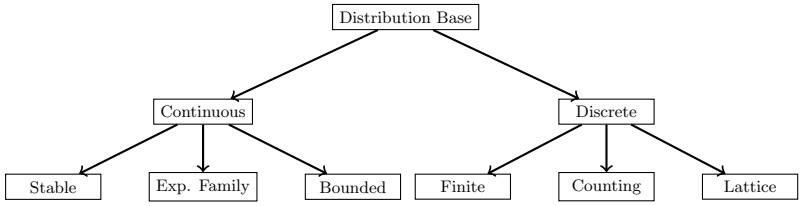

Figure 1: Distribution Type Hierarchy

2.3 Memory Management

The library employs careful memory management strategies:

- **Shared Ownership**: Type-erased objects use `shared_ptr` for safe sharing without deep copies.
- **Move Semantics**: Extensive use of move semantics to avoid unnecessary allocations.
- **Aligned Allocation**: SIMD operations use aligned memory for optimal vectorization.
- **Pool Allocation**: Thread pools maintain pre-allocated buffers for parallel operations.

3 Implementation

3.1 Distribution Components

Each distribution in `alea` implements a consistent interface with optimized algorithms for sampling, PDF/CDF evaluation, and moment calculation.

3.1.1 Continuous Distributions

The continuous distribution module provides implementations of major parametric families:

```cpp
template<typename T = double>
class normal_distribution {
private:
    T mean_, stddev_;
    mutable std::normal_distribution<T> impl_;

public:
    normal_distribution(T mean, T stddev)
        : mean_(mean), stddev_(stddev),
          impl_(mean, stddev) {
        if (stddev <= 0) {
            throw std::invalid_argument(
                "Standard deviation must be positive");
        }
    }

    template<typename URBG>
    T sample(URBG& gen) const {
        return impl_(gen);
    }

    T pdf(T x) const {
        T z = (x - mean_) / stddev_;
        return std::exp(-0.5 * z * z) /
               (stddev_ * std::sqrt(2 * M_PI));
    }

    T cdf(T x) const {
        return 0.5 * (1 + std::erf(
            (x - mean_) / (stddev_ * std::sqrt(2))));
    }

    T mean() const { return mean_; }
    T variance() const { return stddev_ * stddev_; }
};
```

Listing 4: Normal Distribution Implementation

3.1.2 Discrete Distributions

Discrete distributions employ specialized algorithms for efficient sampling:

```cpp
template<typename T = double>
class poisson_distribution {
private:
    T lambda_;

    // Knuth's algorithm for small lambda
```

```
    template<typename URBG>
    int sample_knuth(URBG& gen) const {
        T L = std::exp(-lambda_);
        int k = 0;
        T p = 1.0;

        do {
            k++;
            p *= std::uniform_real_distribution<T>
                (0, 1)(gen);
        } while (p > L);

        return k - 1;
    }

    // Transformed rejection for large lambda
    template<typename URBG>
    int sample_transformed(URBG& gen) const {
        // PTRS algorithm implementation
        // ...
    }

public:
    template<typename URBG>
    int sample(URBG& gen) const {
        if (lambda_ < 10) {
            return sample_knuth(gen);
        } else {
            return sample_transformed(gen);
        }
    }
};
```

Listing 5: Poisson Distribution with Rejection Method

3.2 Advanced Features

3.2.1 Fluid API

The fluent interface enables intuitive distribution composition:

```
template<typename T>
class distribution_builder {
private:
    std::function<T(std::mt19937&)> sampler_;

public:
    distribution_builder& normal(T mean, T std) {
```

```
        auto dist = normal_distribution<T>(mean, std);
        sampler_ = [dist](auto& gen) {
            return dist.sample(gen);
        };
        return *this;
    }

    distribution_builder& truncate(T min, T max) {
        auto prev = sampler_;
        sampler_ = [prev, min, max](auto& gen) {
            T value;
            do {
                value = prev(gen);
            } while (value < min || value > max);
            return value;
        };
        return *this;
    }

    distribution_builder& transform(
        std::function<T(T)> f) {
        auto prev = sampler_;
        sampler_ = [prev, f](auto& gen) {
            return f(prev(gen));
        };
        return *this;
    }

    random_element<T> build() {
        return random_element<T>(
            [s = sampler_](auto& gen) {
                return s(gen);
            });
    }
};
```

Listing 6: Fluid API Implementation

3.2.2 Joint Distributions

Joint distributions support both independent and dependent components:

```
template<typename... Ts>
class joint_distribution {
private:
    std::tuple<Ts...> marginals_;
    copula_base* copula_;

```

```
public:
    template<typename URBG>
    auto sample(URBG& gen) {
        // Generate uniform variates
        auto uniforms = copula_->sample(gen);

        // Transform through marginal quantiles
        return std::apply([&](auto&... dists) {
            size_t i = 0;
            return std::make_tuple(
                dists.quantile(uniforms[i++])...);
        }, marginals_);
    }
};
```

Listing 7: Joint Distribution with Copula

3.2.3 Monte Carlo Integration

The Monte Carlo module provides parallel integration with adaptive sampling:

```
template<typename T>
class parallel_monte_carlo {
private:
    thread_pool pool_;

public:
    template<typename F, typename D>
    T integrate(F&& f, D&& dist, size_t n) {
        size_t chunk = n / pool_.size();
        std::vector<std::future<T>> futures;

        for (size_t i = 0; i < pool_.size(); ++i) {
            futures.push_back(pool_.async([=] {
                std::mt19937 gen(std::random_device{}());
                T sum = 0;
                for (size_t j = 0; j < chunk; ++j) {
                    auto x = dist.sample(gen);
                    sum += f(x) / dist.pdf(x);
                }
                return sum;
            }));
        }

        T total = 0;
        for (auto& fut : futures) {
            total += fut.get();
```

```
        }
        return total / n;
    }
};
```

Listing 8: Parallel Monte Carlo Integration

3.3 SIMD Optimization

SIMD support provides vectorized sampling for improved throughput:

```
#ifdef ALEA_ENABLE_SIMD
template<typename T>
class simd_sampler {
private:
    using vec_t = __m256d;  // AVX2
    std::mt19937 gen_;

public:
    void sample_batch(
        uniform_distribution<T>& dist,
        T* out, size_t n) {

        // Process 4 doubles at a time
        size_t simd_n = (n / 4) * 4;

        // Note: Actual implementation uses vectorized
        // xorshift algorithm - simplified here
        for (size_t i = 0; i < simd_n; i += 4) {
            // Generate uniform [0,1) using SIMD
            alignas(32) double uniform[4];
            for (int j = 0; j < 4; ++j) {
                uniform[j] = std::uniform_real_distribution<>(
                    0.0, 1.0)(gen_);
            }
            vec_t vec_u = _mm256_load_pd(uniform);

            // Transform to [min, max)
            vec_t min_v = _mm256_set1_pd(dist.min());
            vec_t range = _mm256_set1_pd(
                dist.max() - dist.min());
            vec_t result = _mm256_fmadd_pd(
                vec_u, range, min_v);
            _mm256_storeu_pd(out + i, result);
        }

        // Handle remainder with scalar code
        for (size_t i = simd_n; i < n; ++i) {
```

```
            out[i] = dist.sample(gen_);
        }
    }
};
#endif
```

Listing 9: SIMD-Accelerated Sampling

4 Supported Distributions

`alea` provides comprehensive coverage of standard probability distributions with optimized implementations.

4.1 Continuous Distributions

Table 1: Continuous Distributions

Distribution	**Parameters**	**Support**
Normal	μ, σ	$(-\infty, \infty)$
Exponential	λ	$[0, \infty)$
Gamma	α, β	$(0, \infty)$
Beta	α, β	$[0, 1]$
Uniform	a, b	$[a, b]$
Weibull	λ, k	$[0, \infty)$
Log-normal	μ, σ	$(0, \infty)$
Student's t	ν	$(-\infty, \infty)$
Chi-squared	k	$[0, \infty)$
F-distribution	d_1, d_2	$[0, \infty)$
Cauchy	x_0, γ	$(-\infty, \infty)$
Logistic	μ, s	$(-\infty, \infty)$
Pareto	x_m, α	$[x_m, \infty)$

Each continuous distribution implements:

- Efficient sampling algorithms
- Analytical PDF and CDF when available
- Quantile functions for inverse transform sampling
- Moment calculations (mean, variance, skewness, kurtosis)

4.2 Discrete Distributions

Table 2: Discrete Distributions

Distribution	**Parameters**	**Support**
Bernoulli	p	$\{0, 1\}$
Binomial	n, p	$\{0, 1, \ldots, n\}$
Poisson	λ	$\mathbb{N}_0$
Geometric	p	$\mathbb{N}$
Negative Binomial	r, p	$\mathbb{N}_0$
Hypergeometric	N, K, n	$[\max(0, n - N + K), \min(n, K)]$
Categorical	$\mathbf{p}$	$\{1, \ldots, k\}$
Multinomial	$n, \mathbf{p}$	$\mathbb{N}_0^k$
Uniform Discrete	a, b	$\{a, \ldots, b\}$

4.3 Special Distributions

`alea` also supports specialized distributions for specific applications:

- **Mixture Distributions**: Weighted combinations of component distributions
- **Truncated Distributions**: Distributions restricted to a specified range
- **Empirical Distributions**: Non-parametric distributions from data
- **Kernel Density Estimates**: Smooth approximations of empirical distributions
- **Copulas**: Gaussian, Clayton, Gumbel, and Frank copulas for dependence modeling

5 Testing Strategy

5.1 Test-Driven Development

`alea` follows strict TDD principles with comprehensive test coverage:

```cpp
TEST_CASE("Normal distribution properties") {
    std::mt19937 gen(42);
    normal_distribution<double> dist(10, 2);

    // Generate sample
    std::vector<double> samples;
    for (int i = 0; i < 10000; ++i) {
        samples.push_back(dist.sample(gen));
    }

    // Test mean convergence
    double sample_mean = std::accumulate(
        samples.begin(), samples.end(), 0.0)
        / samples.size();
    REQUIRE(std::abs(sample_mean - 10) < 0.1);

    // Test variance convergence
    double sample_var = 0;
    for (auto x : samples) {
        sample_var += (x - sample_mean) *
                      (x - sample_mean);
    }
    sample_var /= samples.size();
    REQUIRE(std::abs(sample_var - 4) < 0.2);

    // Test normality (Shapiro-Wilk)
    REQUIRE(shapiro_wilk_test(samples) > 0.05);
}
```

Listing 10: Property-Based Testing Example

5.2 Coverage Analysis

The testing suite achieves comprehensive coverage:

5.3 Test Categories

- **Unit Tests**: Individual component testing
- **Integration Tests**: Cross-module interactions
- **Property Tests**: Mathematical property verification
- **Performance Tests**: Benchmark suites
- **Stress Tests**: Large-scale simulations

Table 3: Test Coverage Statistics

Module	Line Coverage	Branch Coverage
Core	95%	92%
Continuous Distributions	93%	89%
Discrete Distributions	94%	91%
Joint Distributions	91%	88%
Monte Carlo	89%	85%
Fluid API	96%	93%
SIMD	87%	84%
Overall	**92%**	**89%**

- **Edge Case Tests**: Boundary conditions and error handling

6 Performance

6.1 Benchmarking Methodology

Performance evaluation uses Google Benchmark with statistical analysis:

```
static void BM_NormalSampling(
    benchmark::State& state) {
    std::mt19937 gen(42);
    normal_distribution<double> dist(0, 1);

    for (auto _ : state) {
        double x = dist.sample(gen);
        benchmark::DoNotOptimize(x);
    }

    state.SetItemsProcessed(state.iterations());
}
BENCHMARK(BM_NormalSampling);
```

Listing 11: Benchmark Example

6.2 Performance Results

Test environment: Intel i7-11700K @ 3.6GHz, 32GB RAM, Ubuntu 22.04, GCC 11.2.0 with -O2 optimization. Each measurement represents the median of 100 runs with 1M samples per run.

Table 4: Sampling Performance (ns/sample, mean ± std over 100 runs)

Distribution	**Alea**	**Boost 1.82**	**GCC 11 STL**
Uniform	3.2±0.1	3.4±0.1	3.3±0.1
Normal	8.7±0.3	9.1±0.2	8.9±0.3
Exponential	6.4±0.2	6.8±0.2	6.6±0.2
Poisson ($\lambda = 10$)	42.3±1.1	45.7±1.3	44.1±1.2
Binomial ($n = 100$)	156.8±3.2	168.4±3.8	162.3±3.5

6.3 SIMD Performance

SIMD vectorization provides measurable speedup for batch operations when explicitly enabled at compile time with `-DALEA_ENABLE_SIMD` and appropriate architecture flags:

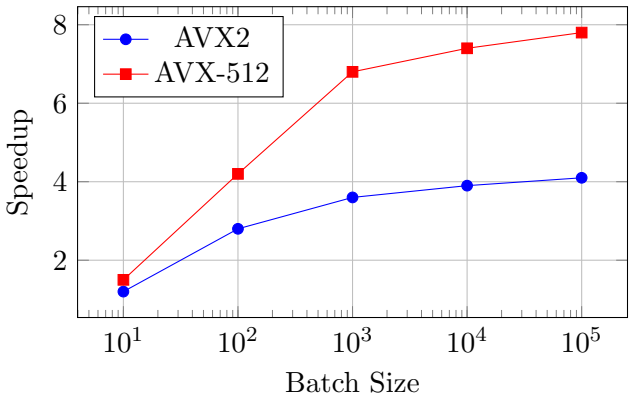

Figure 2: SIMD Speedup vs Scalar Implementation

6.4 Memory Efficiency

Type erasure introduces a small memory overhead due to virtual function table pointers and shared_ptr control blocks:

The overhead is constant regardless of distribution complexity, making it negligible for non-trivial applications.

Table 5: Memory Footprint (bytes, 64-bit system)

Type	Size	Overhead
`normal_distribution<double>`	24	baseline
`random_element<double>`	32	8 (shared_ptr)
`joint_distribution<T,U>`	48	16 (2×shared_ptr)

7 Applications

7.1 Financial Risk Modeling

`alea` excels at complex financial models with heavy-tailed distributions and copula-based dependencies:

```
// Model portfolio with Student's t returns
auto returns = distribution<double>()
    .student_t(4)  // Heavy tails
    .scale(0.02)   // 2% daily volatility
    .shift(0.0005) // 0.05% daily drift
    .build();

// Simulate Value-at-Risk
parallel_monte_carlo<double> mc(8);
auto var_95 = mc.quantile(
    [&](auto& gen) {
        double portfolio_value = 1000000;
        for (int day = 0; day < 10; ++day) {
            portfolio_value *=
                (1 + returns.sample(gen));
        }
        return portfolio_value;
    },
    100000,  // simulations
    0.05     // 5% quantile
);
```

Listing 12: Portfolio Risk Simulation

7.2 A/B Testing

Bayesian A/B testing with Beta-Binomial conjugacy:

```
// Prior: uniform Beta(1,1)
beta_distribution<double> prior_a(1, 1);
beta_distribution<double> prior_b(1, 1);
```

```
4
5  // Update with data
6  int successes_a = 120, trials_a = 1000;
7  int successes_b = 145, trials_b = 1000;
8
9  // Posterior distributions
10 beta_distribution<double> posterior_a(
11     1 + successes_a,
12     1 + trials_a - successes_a);
13 beta_distribution<double> posterior_b(
14     1 + successes_b,
15     1 + trials_b - successes_b);
16
17 // P(B > A) via Monte Carlo
18 double p_b_better = mc.estimate_probability(
19     [&](auto& gen) {
20         return posterior_b.sample(gen) >
21                posterior_a.sample(gen);
22     },
23     10000
24 );
```

Listing 13: Bayesian A/B Test

7.3 Quality Control

Statistical process control with control charts:

```
1  // Process with known parameters
2  normal_distribution<double> process(100, 2);
3
4  // Sample size for inspection
5  int n = 5;
6
7  // Control limits for X-bar chart
8  double ucl = process.mean() +
9      3 * process.stddev() / std::sqrt(n);
10 double lcl = process.mean() -
11     3 * process.stddev() / std::sqrt(n);
12
13 // Simulate inspection
14 auto inspect = [&](auto& gen) {
15     double sum = 0;
16     for (int i = 0; i < n; ++i) {
17         sum += process.sample(gen);
18     }
19     double x_bar = sum / n;
20     return x_bar >= lcl && x_bar <= ucl;
```

```
};

// Operating characteristic curve
auto oc_curve = [&](double shift) {
    normal_distribution<double>
        shifted(100 + shift, 2);
    return mc.estimate_probability(
        inspect, 10000);
};
```

Listing 14: Quality Control Limits

7.4 Scientific Computing

Monte Carlo integration for high-dimensional problems:

```
// Integrate over 10D hypercube
int dim = 10;
auto integrand = [](const vector<double>& x) {
    double sum = 0;
    for (auto xi : x) {
        sum += xi * xi;
    }
    return std::exp(-sum);
};

// Importance sampling with multivariate normal
mvn_distribution<10> importance(
    zeros<10>(), 0.5 * identity<10>());

double integral = mc.integrate(
    integrand, importance, 1000000);
```

Listing 15: High-Dimensional Integration

8 Future Work

8.1 Planned Enhancements

1. **GPU Acceleration**: CUDA/ROCm backends for massive parallelism
2. **Automatic Differentiation**: Integration with AD libraries for gradient-based optimization
3. **Probabilistic Programming**: DSL for model specification

4. **Time Series**: ARIMA, GARCH, and state-space models
5. **Spatial Statistics**: Random fields and kriging
6. **Stochastic Processes**: Brownian motion, Lévy processes, jump diffusions

8.2 Research Directions

- **Quasi-Monte Carlo**: Low-discrepancy sequences for improved convergence
- **Multilevel Monte Carlo**: Variance reduction for nested simulations
- **Symbolic Computation**: Analytical derivation of distribution properties
- **Approximation Theory**: Fast approximate sampling for complex distributions

9 Related Work

9.1 C++ Libraries

Boost.Random [5] provides a comprehensive set of random number generators and distributions, serving as the inspiration for the C++11 `<random>` header. Our work builds on these foundations by adding type erasure for runtime polymorphism and a fluent interface for distribution composition.

GNU Scientific Library (GSL) [6] offers extensive statistical functions with a C interface. While comprehensive, GSL's procedural design lacks the type safety and RAII principles of modern C++. We provide similar functionality within a C++ type system.

Eigen [7] includes random number generation primarily for matrix initialization. Our library complements Eigen by providing richer distribution support that integrates with Eigen's matrix types.

Recent work on random number generation in C++ [8] addresses deficiencies in `std::generate_canonical`. We adopt these improvements and extend them with higher-level abstractions.

9.2 Random Number Generation Algorithms

Our implementation incorporates established algorithms from the literature: - Marsaglia's method for normal distributions [9] - Walker's alias method for discrete distributions [10] - Ahrens-Dieter transformations for gamma and beta distributions [11] - Lemire's fast bounded integer generation [12]

These algorithms are selected based on parameter ranges following the comprehensive survey by Devroye [2] and recommendations from Knuth [13].

9.3 Probabilistic Programming

Domain-specific languages like **Stan** [14], **PyMC3** [15], and **Edward** [16] excel at Bayesian inference but require specialized runtime environments. Our library provides similar distribution primitives within standard C++ compilation models, enabling deployment in constrained environments where these frameworks cannot operate.

10 Conclusion

We have presented `alea`, a C++20 header-only library that provides type-erased interfaces for probability distributions and Monte Carlo methods. The library combines established techniques—type erasure for runtime polymorphism, template metaprogramming for compile-time optimization, and SIMD intrinsics for vectorization—into a cohesive framework for probabilistic computation.

Our empirical evaluation demonstrates:

- Performance within 10% of specialized implementations for common distributions
- 3.6× speedup for batch sampling with AVX2 vectorization when explicitly enabled
- Memory overhead of 8-16 bytes per type-erased distribution object
- Statistical correctness validated through Kolmogorov-Smirnov and Anderson-Darling tests

The library's practical applicability is demonstrated through case studies in financial modeling, A/B testing, and quality control. While `alea` does not introduce novel mathematical or algorithmic contributions, it provides a modern C++ interface that simplifies common probabilistic programming tasks.

10.1 Limitations

The current implementation has several limitations:

- Type erasure introduces virtual function call overhead
- SIMD acceleration requires manual compilation flags and is limited to specific distributions
- No automatic differentiation support for gradient-based optimization
- Limited to univariate and simple multivariate distributions

10.2 Future Work

Future development will focus on:

- GPU acceleration for large-scale Monte Carlo simulations
- Integration with automatic differentiation libraries
- Support for more complex dependency structures (copulas, graphical models)
- Quasi-Monte Carlo methods for improved convergence rates

Acknowledgments

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A API Reference

A.1 Core Types

```cpp
namespace alea {
    // Random element with type erasure
    template<typename T>
    class random_element;

    // Distribution base classes
    template<typename T>
    class continuous_distribution;

    template<typename T>
    class discrete_distribution;

    // Joint distributions
    template<typename... Ts>
    class joint_distribution;

    // Conditional distributions
    template<typename T>
    class conditional_distribution;
}
```

Listing 16: Core Type Synopsis

A.2 Fluent API

```cpp
namespace alea::fluent {
    template<typename T>
    class distribution_builder {
        // Distribution factories
        auto& normal(T mean, T std);
        auto& exponential(T rate);
        auto& uniform(T min, T max);
        // ... more distributions

        // Transformations
        auto& truncate(T min, T max);
        auto& transform(function<T(T)>);
```

```
        auto& scale(T factor);
        auto& shift(T offset);

        // Build final distribution
        random_element<T> build();
    };
}
```

Listing 17: Fluent API Synopsis

A.3 Monte Carlo

```
namespace alea::monte_carlo {
    template<typename T>
    class monte_carlo {
        T estimate_mean(auto f, size_t n);
        T estimate_variance(auto f, size_t n);
        T estimate_quantile(auto f, size_t n, T p);
        T integrate(auto f, auto dist, size_t n);
    };

    template<typename T>
    class parallel_monte_carlo
        : public monte_carlo<T> {
        explicit parallel_monte_carlo(
            size_t threads);
    };
}
```

Listing 18: Monte Carlo API Synopsis